Exchange rate dynamics and the welfare effects of monetary policy in a two-country model with home-product bias

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Abstract

International spillovers and exchange rate dynamics are examined in a two-country dynamic optimizing model that nests Obstfeld and Rogoff (1995) and allows for home-product bias in consumption patterns. Allowing for home bias changes the results in three ways. Wealth transfers associated with net foreign asset positions induce movements in the real exchange rate and produce large short-run and small long-run deviations from consumption-based purchasing power parity. Interest rates, both real and nominal, can differ across countries; home bias is a necessary but not sufficient condition for Dornbusch (1976) type exchange rate overshooting. And the welfare effects of monetary policy depend not only on world demand but also on the expenditure-switching effect of an exchange rate depreciation; expansionary monetary policy is ‘beggar-thy-neighbor’ if individuals have strong preferences for domestic goods.

JEL classification: F3; F4

Keywords: Beggar-thy-neighbor; Purchasing power parity; Dynamic optimizing model; Exchange rate overshooting

“The fact ... that purchasers in each country have a greater familiarity with ... their own country’s products will cause purchasers ... to have some natural preference for the purchase of home products.”

J.E. Meade (1951)
1. Introduction

International macroeconomic models have traditionally incorporated the presumption of a home-product bias in spending. From the early attempts to add international transactions to the Keynesian model (Machlup, 1943; Meade, 1951), to the advent of models with perfect capital mobility (Mundell, 1963, 1968; Fleming, 1962), the presumption has been that foreign trade is a small portion of total economic activity. I follow in this tradition by investigating the effects of home bias on the transmission of monetary policy in a two-country dynamic optimizing model.

Some models have deviated from the presumption of home bias. In the 1970s, in models of the monetary approach to the balance of payments, traded goods produced in different countries were assumed to be perfect substitutes. More recently, individuals in the two-country, sticky-price dynamic optimizing model of Obstfeld and Rogoff (1995) have identical preferences for all goods.

The assumption of identical tastes simplifies the analysis but is somewhat restrictive. Identical goods preferences (and the law of one price) imply that both relative and absolute consumption-based purchasing power parity (PPP) hold at all times. Identical preferences also preclude exchange rate overshooting: interest rates, both real and nominal, are identical across countries, so uncovered interest parity (UIP) implies that after a monetary expansion the nominal exchange rate jumps immediately to its long-run level.

Allowing for a home-product bias makes the model consistent with two aspects of observed exchange rate behavior that macroeconomic models are poor at replicating: the extreme volatility of nominal exchange rates and the existence of long-run deviations from PPP. As home bias increases, Dornbusch (1976) type nominal exchange rate overshooting becomes more pronounced. Moreover, when there is home bias, nominal exchange rates are more volatile than fundamentals such as price levels and money supplies, an empirical regularity which real business cycle models have trouble matching (see Chari et al., 1998).

It is not surprising in a model with home bias and sticky prices that asymmetric changes in money supplies produce short-run deviations from relative PPP, but they also produce small permanen movements in the real exchange rate, a result that is consistent with empirical findings.1 In the short run, a change in relative money supplies causes a current account imbalance. Current accounts must be balanced in the long run, but the short-run imbalance results in a permanent net foreign asset position; monetary policy is not neutral in the long run. The interest payments on the permanent net foreign asset position represent (small) wealth transfers. With home bias, the wealth transfer is spent disproportionately on domestically produced

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1 There is vast empirical evidence that deviations from PPP exist and that convergence to PPP is a slow process; see, for example, the review of Froot and Rogoff (1995).
goods, inducing (small) permanent movements in the real exchange rate and, hence, small permanent deviations from relative PPP.²

Allowing for differences in tastes across countries has implications for welfare analysis. When preferences are identical (and initial net foreign assets are zero), a domestic monetary expansion causes an equal increase in utility throughout the world. With home bias, monetary expansion has the standard ‘beggar-thy-neighbor’ effect. The reason is straightforward. The expenditure-increasing effect on utility is identical across countries and is not affected by the degree of home bias, but the expenditure-switching effect, evident only when there is home bias, increases domestic utility at the expense of foreign utility. With strong enough home bias, the switching effect is greater in magnitude than the increasing effect and foreign utility decreases. Only when tastes are identical does the switching effect drop out; in this case utility in both countries depends only (and identically) on changes in world demand.

The model presented here belongs to what has been termed the ‘new open economy macroeconomics’ that has grown from Obstfeld and Rogoff’s seminal 1995 paper; see Lane (2001) for an excellent survey. Other models, developed simultaneously, that like this one have the law of one price but obviate the assumption of identical tastes, are Ghironi (1998), which allows for biased spending habits across continents in a US-Europe model, and Hau (2000), which allows for nontraded goods; both use a less general specification for real balances and therefore cannot produce exchange rate overshooting.³ The law of one price does not hold in the pricing-to-market models of Tille (2000) and Betts and Devereux (2000). While there is much empirical evidence against the law of one price (Isard, 1977; Engel and Rogers, 1996), Obstfeld and Rogoff (2000) point out that in pricing-to-market models an exchange rate depreciation counterfactually improves a country’s terms of trade. In any case, many of the conclusions from such models are similar to mine. Finally, home bias has been incorporated into the numerical simulations of real business cycle theorists; Chari et al. (1998) assume home bias, but do not spell out its implications.

The paper is organized as follows. In Section 2 the dynamic optimizing model is developed. In Section 3 analytical solutions, obtained by log-linearizing around an initial steady state, are discussed and a numerical example is presented. The welfare implications of home bias are investigated in Section 4. Section 5 concludes.

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² Branson and Henderson (1985) highlight the effect of wealth transfers in a setting with home bias of both assets and goods. Krugman (1990) notes the importance of capital flows and biased consumption on the equilibrium real exchange rate.

³ Corsetti and Pesenti (2001), by separating the elasticity of substitution between domestic and foreign goods from the degree of monopolistic competition, are able to get closed-form solutions for a two-country dynamic optimizing model with logarithmic preferences for real balances by assuming a two-good world with identical tastes, consumption-based PPP, and unitary elasticity of substitution. Taken together these assumptions imply that current account imbalances never occur.
2. A two-country optimizing model with home bias

In this section I develop a two-country optimizing model with home bias and derive the steady-state and short-run changes used to analyze the model. Throughout I assume that the two countries are mirror images of identical size with equal bias for domestically produced goods. The mirror image assumption allows for home bias and has as a special case identical preferences.

2.1. The structural model and initial steady state

The setup follows Obstfeld and Rogoff (1995). There are two equally sized countries (Home and Foreign) inhabited by a continuum of producer/consumers indexed by \( z \in [0,1] \), who each produce a single differentiated product. World population is normalized to one. Prices are sticky. There is no uncertainty except for one-time unanticipated shocks that are either temporary or permanent. Individuals in both countries can freely borrow and lend on world capital markets. The supply side of the model is based on the static closed-economy models of Blanchard and Kiyotaki (1987) and Ball and Romer (1989). For simplicity, it is assumed that output is equal to labor input. The demand side is based on a Dixit and Stiglitz (1977) type consumption index.

The problem of a typical Home resident, \( z \), is presented in Table 1; the typical Foreign resident’s problem is analogous. A Home resident chooses paths for consumption, \( C \); money and bond holdings, \( M \) and \( B \); and effort or output, \( y(z) \), to maximize a time separable utility function with unitary elasticity of intertemporal substitution given by (T1.1). It is assumed that \( \beta \), the subjective discount factor, is strictly between zero and one, and \( \epsilon \), the inverse of the elasticity of substitution between consumption and real balances, is greater than zero. The setup does not preclude exchange rate overshooting because it does not impose logarithmic preferences for real balances (\( \epsilon = 1 \)).

In each period \( t \) a Home resident faces a budget constraint given by Eq. (T1.2),

\[
B_{H,t} + E_B_{F,t} + M_t = (1 + i_{t-1})B_{H,t-1} + (1 + i_{t-1}^*)B_{F,t-1} + M_{t-1} + p(z)y(z) - P_t C_t - P_t T_t
\]

\[
y^d(z) = \frac{1}{2}(c_H(z) + c^*_F(z)) = \frac{1}{2} \left( \frac{p(z)}{P} \right)^{-\theta} \alpha C + \left( \frac{p(z)}{EP^*} \right)^{-\theta} \left( 2 - \alpha \right) C^*
\]

Table 1
A typical home producer/consumer’s problem

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T1.1)</td>
<td>( U_t = \sum_{t=0}^\infty \beta^t \left[ \log C_t + \frac{\chi}{1-\epsilon} \left( \frac{M_t}{P_t} \right)^{1-\epsilon} - \frac{\kappa_s}{2} y(z)^2 \right] )</td>
</tr>
<tr>
<td>(T1.2)</td>
<td>( B_{H,t} + E_B_{F,t} + M_t = (1 + i_{t-1})B_{H,t-1} + (1 + i_{t-1}^*)B_{F,t-1} + M_{t-1} + p(z)y(z) - P_t C_t - P_t T_t )</td>
</tr>
<tr>
<td>(T1.3)</td>
<td>( y^d(z) = \frac{1}{2}(c_H(z) + c^<em>_F(z)) = \frac{1}{2} \left( \frac{p(z)}{P} \right)^{-\theta} \alpha C + \left( \frac{p(z)}{EP^</em>} \right)^{-\theta} \left( 2 - \alpha \right) C^* )</td>
</tr>
</tbody>
</table>

\( ^4 \) The mirror image assumption can easily be relaxed when analyzing numerical solutions.
which takes into account costless international lending and borrowing. It is assumed that the government’s budget is balanced each period, so increases in the money supply with no change in government spending result in lump-sum transfer payments, given by $-T_t$ in Eq. (T1.2). A Home producer, $z$, takes as given world demand for her product, defined as a population weighted average of Home and Foreign demands, given by Eq. (T1.3).

In Table 2, which shows the consumption index and demand functions for Home and Foreign goods, $p(z)$ and $p^*(z)$ are prices of a good $z \in [0,1/2]$ produced in Home and a good $z \in (1/2,1]$ produced in Foreign, respectively; $E$ is the Home currency price of Foreign currency; $P$ is the consumer price index (CPI); and the law of one price is assumed to hold. Home bias is introduced through a parameter in the consumption index, Eq. (T2.1), which is a modification of the Dixit and Stiglitz (1977) constant elasticity of substitution form. Specifically, there is a home-good bias in consumption patterns if at given relative prices the ratio of Home goods consumed to Foreign goods consumed is higher in Home. A Home consumer’s demand functions for Home and Foreign goods, Eqs. (T2.2) and (T2.3), derived by minimizing the cost of one unit of composite consumption taking product prices as given, is shown to depend on the home bias parameter, $\alpha (\alpha \in (0,2))$, and relative prices. The intratemporal substitution elasticity between Home and Foreign goods, $\theta$, is also the price elasticity of demand faced by each producer, and is assumed to be greater than one to ensure that marginal revenue is positive.

From Table 2, the relative demand of a Home consumer for Home goods is

Table 2
A home consumer’s consumption index

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
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</table>
| (T2.1)   | \[C = \left[ \int_0^{1/2} \alpha^{1/3}(c_H(z))^{\theta-1} dz + \int_{1/2}^1 (2-\alpha)^{1/3}(c_F(z))^{\theta-1} dz \right]^{1/\theta} \]
| (T2.2)   | \[c_H(z) = \alpha(p(z)/P)^{-\alpha}C, \ z \in [0,1/2] \]
| (T2.3)   | \[c_F(z) = (2-\alpha) \left[ \frac{E p^*(z)}{P} \right]^{-\theta} C, \ z \in (1/2,1] \]

5 The stocks of nominal bonds, which are denominated in the Home currency, held by Home and Foreign residents entering period $t + l$ are $B_t$ and $B^*_t$, respectively. The nominal interest rates, $i_t$ and $i^*_t$, are defined as the interest rates earned between $t$ and $t + l$. The zero net bond condition imposes $B_t + F_t B^*_t = 0$.

6 More generally, $T_t$ represents lump-sum taxes. See Warnock (1999) for an analysis of the effects of balanced-budget government spending in this setup.

7 The price index may be interpreted as the minimum cost of one unit of consumption and is given by $P = \left( \int_0^\infty \alpha p(z)^{-\theta} dz + \int_0^1 (2-\alpha)[E p^*(z)]^{-\theta} dz \right)^{1/\theta}$. See Obstfeld and Rogoff (1996), pages 222–228, for a complete description of price indices.

8 The range for $\alpha$ was chosen to nest Obstfeld and Rogoff (1995). Any non-negative range can be used without altering the dynamics of the model, but the initial steady-state levels would be affected.
\[ \frac{c_H}{c_F} = \left( \frac{\alpha}{2-\alpha} \right) \left( \frac{p}{Ep^*} \right)^{-\theta}, \]

where \( p \) and \( p^* \) are the prices of the typical Home and Foreign good, respectively. If \( \alpha > 1 \), there is a home bias in consumption. That is, for any given relative price, Home consumers will always demand relatively more Home goods than will Foreign consumers, although for high enough relative prices a Home consumer, even with a home bias, will demand more imported than domestic goods.

Fig. 1, drawn for unitary relative prices, shows the effect of home bias on a consumer’s choice between domestic and imported goods. Consumers with identical preferences for all goods regardless of origin have utility curve \( u_A \) and income-consumption path \( OA \); with unitary relative prices, such consumers demand equal amounts of domestic and imported goods. Consumers with a home bias (\( \alpha = 1.6 \) in this case) have utility curve \( u_B \) and income-consumption path \( OB \); with unitary relative prices, relative demand for domestic goods is \( \alpha/(2 - \alpha) \), or four.

The first-order conditions from a typical Home individual’s dynamic maximization problem are given in Table 3; Foreign individuals have analogous first-order conditions. Equation (T3.1) is the standard first-order consumption Euler equation for unitary elasticity of intertemporal substitution. The consumption-based money demand equation (T3.2) equates the marginal utility of real balances to the opportunity cost in terms of consumption, and reflects the fact that at the margin in period \( t \) individuals must be indifferent between consuming one unit of \( C \) and using the

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9 Since all individuals in a country are symmetric and choose the same price and output in equilibrium, the \( z \) denotation can be dropped. The price and output of the typical Home good are denoted by \( p \) and \( y \), respectively.
First-order conditions

\[(T3.1)\quad C_{t+1} = \beta C_t (1 + r_t)\]

\[(T3.2)\quad \frac{M_t}{P_t} = \left[ \chi C_t \frac{1 + i_t}{i_t} \right]^{1/\varepsilon}\]

\[(T3.3)\quad y_t^{\theta + 1} = \left( \frac{\theta - 1}{\theta \kappa_t} \right)^{\theta \varepsilon} \left[n \alpha C_t + (1 - n)(2 - \alpha^*) \left( \frac{E_t P_t}{P_t} \right)^{\theta} \right]^{\theta}\]

\[(T3.4)\quad 1 + r_t = \frac{Q_{t+1}}{Q_t} (1 + r_t^*)\]

same funds to raise cash balances. Equation (T3.3) is the labor-leisure tradeoff: the marginal utility of the additional revenue earned from producing an extra unit of the Home good must equal the marginal disutility of the needed effort. Equation (T3.4) is a real uncovered interest parity condition—both real and nominal uncovered interest parity hold in this perfect foresight model—that captures the dynamics of the real exchange rate, \(Q_t\), defined as \(E_t P_t^* / P_t\).\(^{10}\) (As defined, an increase in \(Q\) is a real depreciation for the Home currency.)

Steady-state relations are presented in Table 4. In solving for the steady state it is assumed that all exogenous variables are constant, which, in turn, implies a constant steady-state consumption level and, by the consumption Euler equation (T3.1), a constant steady-state real interest rate, given by Eq. (T4.1). In the steady state the intertemporal budget constraints are given by Eqs. (T4.2) and (T4.2\(^*\)). It is assumed

\[\text{Table 4}\]

Steady state relations

\[(T4.1)\quad \bar{\rho} = \frac{1 - \beta}{\beta}\]

\[(T4.2)\quad \bar{C} = \bar{B} + \bar{\rho} \frac{\bar{Y}}{\bar{P}}\]

\[(T4.2^*)\quad \bar{C}^* = \frac{\rho \bar{B} \bar{P^*}}{\bar{P}} + \frac{\rho^* \bar{Y}^*}{\bar{P^*}}\]

Initial steady state levels

\[(T4.3)\quad \bar{y}_0 = \bar{Y}_0 = \left( \frac{\theta - 1}{\theta \kappa} \right)^{1/2} = \bar{C}_0 = \bar{C}_0^*\]

\[(T4.4)\quad \bar{M}_0 = \bar{M}_0^* \left( \frac{\bar{X}}{1 - \bar{\rho}} \right)^{1/\varepsilon} = \left( \frac{\bar{X}}{1 - \beta} \right)^{1/\varepsilon} \bar{C}_0^{1/\varepsilon}\]

\(^{10}\) The Home real interest rate and nominal UIP condition in this perfect foresight model are given by

\[1 + i_t = \frac{P_{t+1}}{P_t} (1 + r_t)\] and \[1 + i_t = \frac{E_t}{E_t^*} (1 + i_t^*).\]
in the initial steady state net foreign assets are zero \((B_0 = B_0^* = 0)\).\(^{11}\) From (T4.2) and (T4.2\(^*\)), in the initial steady state consumption equals output, and is given by (T4.3), which follows from the labor-leisure tradeoff (T3.3). Initial steady state real balances are given by Eq. (T4.4).

To analyze the effects of changes in money supplies on Home and Foreign variables, the model is linearized around the initial steady state. The shocks are level shifts that occur in period \(t\) and are temporary (lasting one period) or permanent. In the policy experiments conducted in this paper, one-time unanticipated shocks to the money supply push the economy to a new steady state.

Price stickiness is introduced by assuming producers must set their prices before they observe the shock: for the first period of a shock, \(t\), product prices are constant, but thereafter, in periods \(t + 1\) and beyond, product prices fully adjust and a new steady state is reached. There are, thus, two types of deviations from the initial steady state. Long-run deviations are those consistent with flexible product prices and are changes from the initial to a new steady state. These long-run steady-state changes are denoted by tildes; for any variable \(X\), \(\tilde{X}/X_0\), where \(X_0\) is the initial steady-state value. In the short run when product prices are fixed, the economies are not on a steady-state path. Short-run deviations from the initial steady state are denoted by hats; for any variable \(X\), \(\hat{X}/X_0\), where \(t\) is the period of the shock.

### 2.2. Long-run steady-state changes

Long-run equations for cross-country differences are presented in Table 5. Log-linearizing the Home CPI—found in footnote 7—and its Foreign counterpart around the initial steady state, and using the definitions of the real exchange rate and terms of trade, \(\tau\), yields Eq. (T5.1).\(^{12}\) When preferences are identical \((\alpha = 1)\), the long-run real exchange rate is constant and PPP holds. The log-linearized versions of (T1.3) and its Foreign counterpart, interpreted as world demand schedules for typical home and foreign products, give Eq. (T5.2), which clearly shows how the bias parameter,

<table>
<thead>
<tr>
<th>Table 5</th>
<th>Long-run equations—cross-country differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T5.1)</td>
<td>( \tilde{Q} = (1-\alpha)\tilde{\tau} )</td>
</tr>
<tr>
<td>(T5.2)</td>
<td>( \tilde{y}-\tilde{y}^* = -\alpha(2-\alpha)\theta\tilde{\tau} + (\alpha-1)(\tilde{C}-\tilde{C}^*) )</td>
</tr>
<tr>
<td>(T5.3)</td>
<td>( (\theta + 1)(\tilde{y}-\tilde{y}^<em>) = (\alpha-\theta-1)(\tilde{C}-\tilde{C}^</em>)-(2-\alpha)(\alpha-1)\theta\tilde{\tau} )</td>
</tr>
<tr>
<td>(T5.4)</td>
<td>( (\tilde{C}-\tilde{C}^<em>) = \frac{2rd\tilde{B}}{\tilde{y}_0} + (2-\alpha)\tilde{\tau} + (\tilde{y}-\tilde{y}^</em>) )</td>
</tr>
<tr>
<td>(T5.5)</td>
<td>( \tilde{C}-\tilde{C}^* = (\tilde{C}-\tilde{C}^*) + (\tilde{Q}-\tilde{Q}) )</td>
</tr>
<tr>
<td>(T5.6)</td>
<td>( (\tilde{M}-\tilde{M}^<em>)-(\tilde{P}-\tilde{P}^</em>) = \frac{1}{\epsilon}(\tilde{C}-\tilde{C}^*) )</td>
</tr>
</tbody>
</table>

\(^{11}\) Note that the mirror image assumption and the assumptions made to get an initial, closed form steady state together imply that \(p_0 = E_0\delta p_0^*\), \(p_0 = \tilde{P}_0\) and \(p_0^* = \tilde{P}_0^*\).

\(^{12}\) An increase in \(\tau\), which is defined as \(p/p^*E\), represents an improvement in Home’s terms of trade.
\( \alpha \), affects relative demand. With home bias, spending differentials and changes in the terms of trade affect demand. Spending differentials matter more as home bias becomes more pronounced, and not at all if tastes are identical across countries. The terms of trade influence demand less as \( \alpha \) increases: if citizens have an inherent preference for domestic goods, relative prices matter somewhat less.

Linearized versions of Eq. (T3.3) and its foreign counterpart, which describe the optimal flexible-price output levels, give Eq. (T5.3), which shows that long-run supply differentials are affected by differences in consumption and—if preferences are biased—changes in the terms of trade. An increase in relative consumption makes Home individuals want to enjoy relatively more leisure; relative supply falls. An improvement in Home terms of trade reduces relative supply if \( \alpha > 1 \).

Linearizing (T4.2) and (T4.2\*), the intertemporal budget constraints, around the initial steady state yields Eq. (T5.4), an equation for long-run cross-country consumption differentials. The effect of changes in the terms of trade on consumption differentials is maximized when \( \alpha = 1 \): relative income changes matter most when imported goods carry heavy weights in the consumption basket. Subtracting the log-linearized versions of consumption Euler equations, (T3.1) and its foreign counterpart, yields Eq. (T5.5), which provides one link between the short and long runs, and shows that consumption differentials are permanent after a shock only if the real exchange rate jumps immediately to its long-run level.

Subtracting the long-run linearized versions of the money demand equations, (T4.4) and its foreign counterpart, gives Eq. (T5.6), which uses the fact that across steady states real interest rates and inflation rates do not change. Note that money demand depends on consumption, as in many other intertemporal monetary models. If \( \epsilon = 1 \), the long-run change in relative consumption equals the long-run change in relative real balances; in the more general case of \( \epsilon > 1 \), the change in relative consumption is greater than the change in real balances.

### 2.3. Short-run deviations from the initial steady state

Cross-country differences of short-run deviations from the initial steady state are presented in Table 6. In the short run nominal product prices, \( p \) and \( p^* \), are fixed.

<table>
<thead>
<tr>
<th>Table 6</th>
<th>Short-run equations—cross-country differences</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T6.1) ( \dot{Q} = (\alpha - 1)\dot{E} )</td>
<td></td>
</tr>
<tr>
<td>(T6.2) ( \dot{y} - \dot{y}^* = \alpha(2 - \alpha)\theta\dot{E} + (\alpha - 1)(\dot{C} - \dot{C}^*) )</td>
<td></td>
</tr>
<tr>
<td>(T6.3) ( \dot{C} - \dot{C}^* = \frac{2dB}{\dot{y}_0} + (\dot{y} - \dot{y}^*) - (2 - \alpha)\dot{E} )</td>
<td></td>
</tr>
<tr>
<td>(T6.4) ( (\dot{M} - \dot{M}^<em>) - (2 - \alpha)\dot{E} = \frac{1}{\epsilon}(\dot{C} - \dot{C}^</em>) - \frac{1}{\rho\epsilon}(\dot{E} - \dot{E}) )</td>
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</tr>
<tr>
<td>(T6.5) ( \dot{p} - \dot{p}^* = \frac{\dot{Q} - \dot{Q}}{1 - \beta} )</td>
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</table>
so changes in CPIs depend solely on changes in the exchange rate. Equation (T6.1) highlights two implications of home bias for exchange rates. As home bias increases, the correlation between real and nominal exchange rates increases, and nominal exchange rates become much more volatile than price indices.\(^{13}\)

With preset prices, the equation equating marginal revenue and marginal cost, (T3.3), need not hold. Short-run output is demand-determined and short-run output differentials are given by Eq. (T6.2). As in the long run, spending differentials affect output differentials only if there is home bias and more spending stays at home. Note, too, that the demand-switching effect of nominal exchange rate changes is maximized when \(\alpha = 1\) and decreases as \(\alpha\) increases.

Equations for Home and Foreign current accounts yield Eq. (T6.3), which shows the saving (or borrowing) that arise from imbalances in consumption and output.\(^{14}\) Log-linearizing the money demand equations, (T3.2) and its foreign counterpart, and noting that all \((t + 1)\) subscripts are steady-state changes (tildes) and all \(t\) subscripts are short-run changes (hats), gives Eq. (T6.4), which shows the relationship between the short-run real interest rate differential and real exchange rate dynamics. Equation (T6.5), a real interest parity condition, shows the relationship between real interest rate differentials and real exchange rate dynamics.

### 3. The dynamic responses to monetary shocks

In this section I consider an unexpected permanent monetary shock. The equations that form the basis for solving the model—long-run semi-reduced form equations and the MM and GG schedules—are presented along with a graphical example; analytical solutions are discussed; and a numerical example is presented.

#### 3.1. Long-run semi-reduced form equations and the MM and GG curves

Table 7 contains cross-country differences of long-run semi-reduced form equations that hold for monetary shocks and show the effects of net foreign asset shocks on relative consumption (T7.1), relative output (T7.2) and the terms of trade (T7.3).\(^{15}\)
Although care must be taken when interpreting these partial relationships, some intuition can be gleaned from them. Equation (T7.1) shows that a wealth transfer increases relative consumption, but by less than the change in relative net foreign asset positions: Home individuals take advantage of the increased wealth to both increase consumption and reduce their work effort. From Eq. (T7.3), an increase in Home relative consumption, caused perhaps by a wealth transfer, enables Home individuals to work less and increases the long-run relative price of Home products (the terms of trade).

Table 8 contains the equations for the MM and GG curves, which show relationships between short-run changes in the nominal exchange rate and relative consumption. The MM curve, which is obtained using Eqs. (T5.6) and (T6.4), shows how relative consumption changes, through their effect on relative money demand, affect the exchange rate. The GG curve, Eq. (T8.2), obtained using Eqs. (T6.2), (T6.3), and (T7.1), shows the currency depreciation—and therefore the increase in relative output—needed to justify an increase in relative consumption.

The MM and GG curves are shown graphically in Fig. 2. From Eqs. (T8.1) and (T8.2), as $\alpha$ increases (or $\epsilon$ increases or $\theta$ decreases), the MM curve becomes steeper; when there is home bias consumer prices are relatively insulated, so an increase in relative consumption has a greater effect on relative real balances and therefore a greater effect on the exchange rate. The GG curve flattens as $\alpha$ (or $\theta$) increases; as preferences become more biased, the expenditure switching effect is greater and relative consumption increases more for a given depreciation.

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### Table 7

Long-run semi-reduced forms—cross-country differences

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T7.1) $$(1-\alpha + \theta)(\hat{C} - \hat{C}^*) = \theta(2-\alpha)\hat{Q} + (\theta + 1)\frac{rdB}{y_0}$$</td>
<td></td>
</tr>
<tr>
<td>(T7.2) $$(1 + \theta)(\hat{y} - \hat{y}^<em>) = -\theta(2-\alpha)(\hat{C} - \hat{C}^</em>) - (2-\alpha)(\alpha-1)\theta\bar{\tau}$$</td>
<td></td>
</tr>
<tr>
<td>(T7.3) $$\bar{\tau} = \frac{\alpha(\hat{C} - \hat{C}^<em>)}{(2-\alpha)(\alpha \theta + 1)} = \frac{-\alpha(\hat{y} - \hat{y}^</em>)}{(2-\alpha)(1-\alpha + \alpha \theta)}$$</td>
<td></td>
</tr>
</tbody>
</table>

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### Table 8

**MM and GG curves**

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T8.1) <strong>(MM)</strong> $\hat{E} = -\frac{\gamma_0(1 + \rho \epsilon)}{\gamma_0 \gamma_1 + \alpha(1-\alpha)\gamma_4} (\hat{C} - \hat{C}^<em>) + \frac{\gamma_0(1 + \rho \epsilon)\epsilon}{\gamma_0 \gamma_1 + \alpha(1-\alpha)\gamma_4} (\hat{M} - \hat{M}^</em>)$</td>
<td></td>
</tr>
<tr>
<td>(T8.2) <strong>(GG)</strong> $\hat{E} = \frac{\gamma_0(\gamma_4 + \gamma_3) - \alpha \gamma_4}{\alpha(1-\alpha)\gamma_4 + \gamma_0 \gamma_6} (\hat{C} - \hat{C}^*)$</td>
<td></td>
</tr>
</tbody>
</table>

---

16 Definitions for the $\gamma_i$ in Table 8 and all tables that follow are in the appendix.

17 Note that in order to incorporate long-run changes of the real exchange rate into the curves, Eqs. (T5.5) and (T7.3) are also used in the derivations.
Fig. 2. Short-run changes from an unanticipated permanent increase in Home money supply. Pre-shock equilibrium is at the origin. Primed, thick curves and the post-shock equilibrium A are drawn for \( \alpha > 1 \); unprimed curves and point B are drawn for \( \alpha = 1 \).

The short-run effects of a permanent increase in relative money supply (\( \dot{M} - \dot{M}^{*} > 0 \)) on the nominal exchange rate and relative consumption are also shown in Fig. 2. When preferences are biased towards domestically produced goods, a given increase in relative money supply leads to a larger exchange rate depreciation and a larger change in relative consumption. The initial equilibrium is at the origin; absent a policy shock the exchange rate and consumption differential are constant.

From Eq. (T8.1), the vertical intercept of \( \mathbf{M} \mathbf{M} \) is the amount of the permanent increase in the relative money supply, \( \ddot{M} - \ddot{M}^{*} \), if preferences are identical, but is greater than \( \ddot{M} - \ddot{M}^{*} \) when there is home bias. Thus, in this model the nominal exchange rate is more volatile than relative money supplies when there is home bias. Moreover, since \( \mathbf{G} \mathbf{G} \) is flatter as preferences become more biased towards domestic goods, relative consumption increases more.

3.2. Analytical solutions

It is straightforward to solve the model for country levels after first solving for cross-country differences and world weighted averages. To solve for cross-country differences, use the \( \mathbf{M} \mathbf{M} \) and \( \mathbf{G} \mathbf{G} \) curves to get solutions for \( \ddot{E} \) and \( \dot{C} - \dot{C}^{*} \), which can then be used in Eq. (T6.2) to get \( \ddot{y} - \ddot{y}^{*} \) and in Eq. (T6.3) to get \( \ddot{dB}/\ddot{y}_{0} \). Armed with \( \ddot{dB}/\ddot{y}_{0} \), Table 7 yields long-run solutions.

Table 9 provides world weighted averages for an unexpected permanent change in world money supply.\(^{18}\) Long-run world neutrality of money holds: in the long-

\[^{18}\] The world weighted average for any variable \( X \) is given by \( X^{w} = \frac{1}{2}X + \frac{1}{2}X^{*} = \frac{1}{2}(X + X^{*}). \)
run, world money supply determines world price levels (T9.2) but does not affect world output or consumption (T9.1). Equations (T9.3) and (T9.4)—derived using a world consumption Euler, $\hat{C}^W = \hat{C}^W + (1 - \beta)\hat{p}^W$—show the importance of the interest elasticity of money demand, approximately equal to $1/\epsilon$, in determining the magnitude of short-run changes in real variables. As money demand becomes more interest inelastic (i.e., as $\epsilon$ increases), real interest rates fall more—and, hence, demand and output increase more—for a given increase in world money supply.

Analytical solutions for country levels are given in the appendix. For ease of exposition, the case of a permanent increase in the Home money supply is discussed, although solutions are written for any permanent change in relative money supplies.

### 3.2.1. Short-run solutions: the expenditure-increasing and expenditure-switching effects

Short-run changes are most easily expressed as the sum of two components, the expenditure-increasing effect of a change in world demand, $\hat{C}^W$, given by Eq. (T9.3), and the expenditure-switching effect of a change in the nominal exchange rate, $\hat{E}$, given by $\hat{E} = \frac{\epsilon (1 + \hat{p}^W)\gamma}{\gamma_{11}}(\tilde{M} - \tilde{M}^*)$. For a given increase in Home money supply, the nominal exchange rate depreciates more the greater is the degree of home bias; when imported goods carry little weight in consumption baskets, consumer prices are insulated and the exchange rate must adjust more. Solutions for short-run changes in real Home variables are presented in Table 10. For short-run changes in Foreign

### Table 9
Money supply shock: world weighted averages

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>(T9.1)</td>
<td>$\bar{y}^w = \bar{C}^w = 0$</td>
</tr>
<tr>
<td>(T9.2)</td>
<td>$\bar{p}^w = \bar{M}^w$</td>
</tr>
<tr>
<td>(T9.3)</td>
<td>$\hat{y}^w = \hat{C}^w = \frac{1 + \hat{p}^w}{1 + \hat{p}^w} \bar{M}^w$</td>
</tr>
<tr>
<td>(T9.4)</td>
<td>$\hat{r}^w = -\frac{1 + \hat{p}^w}{\beta(1 + \hat{p}^w)} \bar{M}^w$</td>
</tr>
</tbody>
</table>

### Table 10
The short-run real effects of changes in world demand and relative prices

<table>
<thead>
<tr>
<th>Equation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T10.1)</td>
<td>$\bar{C} = \bar{C}^w + \frac{\hat{E}^w}{\gamma_{10}}\frac{\gamma}{2\gamma_{10}}$</td>
</tr>
<tr>
<td>(T10.2)</td>
<td>$\bar{y} = \bar{C}^w + \frac{\hat{E}^w}{\gamma_{10}}\frac{\gamma}{2\gamma_{10}}(2 - \alpha)\theta\gamma_{10} + (\alpha - 1)\gamma_k$</td>
</tr>
<tr>
<td>(T10.3)</td>
<td>$\hat{r} = -\frac{\hat{C}^w}{\beta} \frac{\hat{E}^w}{\gamma_{10}}\frac{\gamma}{2\gamma_{10}}\gamma_{13} + \frac{\alpha(\theta + 1)\gamma_{12}}{4\beta^2\gamma_{10}\gamma_{13}}$</td>
</tr>
</tbody>
</table>
variables, the expenditure-increasing effects of increased world demand are identical to Home’s, but the expenditure-switching effects have the opposite sign.

For Home consumption, Eq. (T10.1), the expenditure-switching effect increases with $\alpha$; as home bias increases, a smaller portion of Home’s consumption basket becomes more expensive, and Home residents are able to increase consumption even more. Similarly, a smaller portion of Foreign’s consumption basket becomes cheaper, so Foreign residents cannot increase consumption as much. For Home output, Eq. (T10.2), the expenditure-switching effect is maximized when preferences are identical; an exchange rate depreciation has the greatest effect on demand for Home goods when they figure prominently in Foreign consumption baskets. For plausible parameter values, the switching effect on output is greater than the effect of increased world demand, so Foreign output falls. Finally, for the Home real interest rate, Eq. (T10.3), there is no switching effect when preferences are identical, so the drop in real interest rates is identical across countries. The switching-effect on real interest rates becomes greater as home bias increases, causing Home’s real interest rate to fall more, and Foreign’s to fall less.

3.2.2. Long-run solutions: the role of wealth transfers

Long-run changes in real variables are easily expressed as functions of the permanent wealth transfer associated with the interest payments on the net foreign asset position, which is given by $\frac{rdB}{\gamma_0} = (\bar{M} - \bar{M}^*) \frac{(1 - \alpha/2)\bar{r}e(1 + \bar{r}e)\gamma_0\gamma_{12}}{\gamma_{11}}$. Current account imbalances and the associated wealth transfers are greater when there is a large disparity in output, which, from the discussion in the previous section, occurs when home bias is small. When home bias is severe, current account imbalances are small because the decrease in import prices does not induce sufficient purchases. Long-run changes in output, consumption, and the real exchange rate are presented in Table 11.

The wealth transfer associated with a net foreign asset position enables Home residents to work less, Eq. (T11.1), and consume more, Eq. (T11.2), than their Foreign counterparts. Along the lines of Krugman (1990), when preferences are biased the permanent wealth transfer is spent disproportionately on domestic goods, inducing a small permanent change in the real exchange rate (T11.3) that—as will

<table>
<thead>
<tr>
<th>Table 11</th>
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<tbody>
<tr>
<td>The long-run real effects of wealth transfers</td>
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</tbody>
</table>

<table>
<thead>
<tr>
<th>Equation</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T11.1)</td>
<td>$\bar{y} = -\bar{y}^* = -\frac{rdB}{2\bar{y}_0}$</td>
</tr>
<tr>
<td>(T11.2)</td>
<td>$\bar{C} = -\bar{C}^* = \frac{rdB}{2\bar{y}_0} \left(\alpha \theta + 1\right)$</td>
</tr>
<tr>
<td>(T11.3)</td>
<td>$\bar{Q} = -\frac{rdB \alpha(\alpha - 1)}{2\bar{y}_0(2 - \alpha)(1 + \alpha(\theta - 1))}$</td>
</tr>
</tbody>
</table>
be shown in the next section—increases with the degree of bias. Moreover, from Eq. (T5.1), there is a permanent improvement in Home’s terms of trade.

3.3. Numerical solutions

Fig. 3(a)–(i) show, for different degrees of home bias, the effect of a 1% permanent increase in Home money supply. In each graph in Fig. 3, the scale on the x-axis is 100 times $\alpha$; that is, $\alpha = 1$ (identical preferences) is at the tick mark labeled ‘100’ and $\alpha$ varies from 0.01 to 1.99 along the entire x-axis. Home bias is in the area where $\alpha$ is greater than one, while foreign bias (or a preference for foreign goods)
is where $\alpha$ is less than one. Units for $y$-axes are percent changes in the indicated variables.

Parameters other than $\alpha$ are fixed at levels taken from the literature ($\theta = 6$, $\epsilon = 9$, and $\beta = 0.95$).\(^{19}\) It should be noted that the preferences in this model are such that $\epsilon$ determines both the consumption elasticity and the interest elasticity of money demand. Since empirical estimates put the consumption elasticity of money demand at or below 1 (Mankiw and Summers, 1986) and the interest elasticity of money demand at 0.1 (Koenig, 1990), if $\epsilon$ is chosen based on estimates of the consumption elasticity, the interest elasticity is too high.\(^{20}\) To date in the literature, those who have not imposed logarithmic preferences for real balances have presumed $\epsilon > 1$.

A permanent Home monetary expansion causes a short-run nominal exchange rate depreciation (Fig. 3(a)). If there is home bias, and as long as the consumption elasticity of money demand is not equal to 1, the short-run depreciation exceeds the long-run depreciation and is larger than the change in money supply; that is, Dornbusch-type overshooting occurs, and the overshooting is greater the greater is home bias.\(^{21}\) The expenditure-switching effect of the short-run depreciation results in a sharp, temporary increase in Home output (Fig. 3(c)). Movements in consumption are much less pronounced (Fig. 3(h)), because consumption is smoothed by running a current account surplus (Fig. 3(e)). Since product prices are fixed in the short-run and, hence, short-run changes in CPIs are determined solely by the share of imported goods in total expenditure and the change in the exchange rate, greater home bias implies less short-run movement in CPIs (Fig. 3(b)).

The short-run current account surplus results in a permanent net foreign asset position that is the key to the long-run non-neutrality of money. As was shown in Table 11, the interest payments on the net foreign asset position—and, to some extent, the degree of home bias—determine the (small) long-run changes in output (Fig. 3(i)) and consumption (Fig. 3(f)). The degree of home bias plays a much more important role in determining the path of the real exchange rate: if preferences are biased, the small permanent wealth transfers are spent disproportionately on domestic goods, resulting in small permanent deviations from PPP that increase with home bias (Fig. 3(g)). In the long run, consumer prices are little affected by the degree of home bias (Fig. 3(b)).

The results described in this section are reassuringly familiar. The effects of monetary policy in the dynamic optimizing model with home bias are not dissimilar to those of Mundell and Fleming. This should not be too surprising, for the reduced-


\(^{20}\) This point is also made by Betts and Devereux (2000).

\(^{21}\) That identical tastes ($\alpha = 1$) or logarithmic preferences for real balances ($\epsilon = 1$) precludes exchange rate overshooting is illustrated in the following expression $E = \frac{E_1}{1 + r} = \frac{1}{1 + r(\alpha - 1)} + \frac{r(\epsilon - 1)(\alpha - 1)}{1/r}$. A similar condition can be derived from a modified Dornbusch (1976) model with a short-run supply response and a money demand equation in which money balances are deflated by the CPI instead of the price of domestic output and the activity variable is output in terms of consumption units.
form equations postulated in ad hoc models are very similar to the first-order conditions derived in optimizing models.

4. The welfare effects of monetary policy

The infinite-period discounted sum of changes in Home and Foreign utility (disregarding liquidity effects) due to changes in consumption and output is given by:

\[
\frac{dU}{\theta} = \frac{\dot{C}_w}{\theta} + \frac{1}{4\theta} \sum_{i=1}^{13}[\theta y_{ls} - (\theta - 1)[2 - \alpha(\alpha - 1)]]
\]

\[
\frac{dU^*}{\theta} = \frac{\dot{C}_w}{\theta} - \frac{1}{4\theta} \sum_{i=1}^{13}[\theta y_{ls} - (\theta - 1)[2 - \alpha(\alpha - 1)]]
\]

The first term is the (identical) expenditure-shifting effect of increased world demand. Due to monopolistic distortions, initial output is suboptimally low; the increased world demand resulting from expansionary monetary policy increases welfare. If preferences are identical, the expenditure-switching effect (the second term) is zero; absent an expenditure-switching effect, the utility effects of expansionary monetary policy are identical across countries. In the more general case with a home-product bias, the expenditure-switching effect allows Home individuals to gain more utility as consumption rises relative to effort, but this increase is at the expense of Foreign individuals. As can be shown, a small degree of home bias gives the familiar beggar-thy-neighbor result. In this framework in which utility is gained from consumption but reduced with effort (i.e., output), monetary policy is ‘beggar-thy-neighbor’ if individuals have strong preferences for domestically produced goods. If preferences are identical, as in the Obstfeld and Rogoff (1995) model, both countries gaining equally from a monetary expansion.22

5. Conclusions

The model presented in this paper fully nests Obstfeld and Rogoff (1995) while allowing for a home-product bias in consumption. Allowing for home bias enables richer analysis of the welfare implications of monetary policy and has implications for exchange rate determination.

To show the implications home bias has for the welfare effects of monetary policy shocks, changes in utility are divided into two components, the shifting effect of increased world demand and the switching effect of an unexpected exchange rate depreciation. The shifting effect is identical across countries and is not affected by the degree of home bias. The switching effect, apparent only when there is home bias, increases Home utility at the expense of Foreign utility. With enough home bias, monetary policy is beggar-thy-neighbor: the switching effect is great enough

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22 Monetary policy can be beggar-thyself in a world in which consumers have a foreign-good bias.
that Foreign utility falls. Conversely, if individuals prefer imported goods, monetary policy can be beggar-thyself. This division provides a clear explanation of Obstfeld and Rogoff’s welfare result: without home bias there is no switching effect, thus utility in both countries depends only (and identically) on changes in world demand.

The model with home-product bias has a number of implications for exchange rate determination. Not surprisingly, in a model with sticky prices and different consumption baskets across countries, there are short-term deviations from PPP. This model also produces small, permanent deviations from PPP. As argued in Krugman (1990), if preferences are biased, wealth transfers affect the real exchange rate. In my model any asymmetric shock results in a temporary current account imbalance, a permanent net foreign asset position, and, hence, permanent wealth transfers and permanent deviations from PPP. Also, with home bias there is Dornbusch (1976) type overshooting and increased volatility of real and nominal exchange rates.

The model is decidedly simple in a number of respects. Only monetary shocks are investigated here; the effects of government spending and productivity shocks in the same framework are analyzed in Warnock (1999). The absence of investment and the simplicity of the supply process preclude the model from replicating observed patterns in output. Price dynamics would have to be considerably richer, perhaps in the form of Calvo (1983) pricing, if one were to bring the model to the data. The assumption of mirror-image countries is easily relaxed if one were content with numerical solutions.

Acknowledgements

This article is based on chapter 2 of my dissertation at the University of North Carolina. I thank Stanley Black and Dale Henderson for guidance; Cedric Tille and an anonymous referee for helpful comments; and participants at the UNC Money/Macro Seminar, the 1997 Southeastern Economic Theory and International Economics Conference, the 2000 Western Economic Association Conference, and the workshop at the International Finance Division of the Board of Governors of the Federal Reserve System. Remaining errors are my own. The views in this paper are solely the responsibility of the author and should not be interpreted as reflecting the views of the Board of Governors of the Federal Reserve System or of any other person associated with the Federal Reserve System.
Appendix A. Analytical solutions

\[ \dot{E} = \frac{\varepsilon (\bar{M} - \bar{M}^*)}{\gamma_1} (1 + \Re) y_0 \gamma_{10}, \]

(A.1)

\[ \ddot{E} = \frac{\varepsilon (\bar{M} - \bar{M}^*)}{\gamma_5} (1 + \Re (2 - \alpha)) y_0 \gamma_{10} - r \gamma_6 (y_7 \gamma_{10} + \alpha y_2 \gamma_6), \]

(A.2)

\[ \ddot{\tau} = \frac{\alpha \Re (\theta + 1) y_0 \gamma_{12}}{2 \gamma_1 \gamma_{13}}, \]

(A.3)

\[ \frac{dB}{\dot{y}_0} = \frac{(\bar{M} - \bar{M}^*) (1 - \alpha / 2) \varepsilon (1 + \Re) y_0 \gamma_{12}}{\gamma_1}, \]

(A.4)

\[ \ddot{p} = - \dot{p}^* = \frac{(\bar{M} - \bar{M}^*) (2 - \alpha) \varepsilon y_0 \gamma_{10}}{2 \gamma_8 \gamma_{11}}, \]

(A.5)

\[ \ddot{p} = \ddot{M}^w + (\bar{M} - \bar{M}^*) \left( \frac{1}{2} - \frac{(1 + \alpha \theta) y_5 \gamma_6 \gamma_{12}}{4 \gamma_8 \gamma_{11} \gamma_{13}} \right), \]

(A.6)

\[ \ddot{p}^* = \ddot{M}^w - (\bar{M} - \bar{M}^*) \left( \frac{1}{2} - \frac{(1 + \alpha \theta) y_5 \gamma_6 \gamma_{12}}{4 \gamma_8 \gamma_{11} \gamma_{13}} \right), \]

(A.6*)

\[ \ddot{y} = \frac{1 + \Re}{(1 + \bar{r})} \bar{M}^w - \frac{\varepsilon y_5 (2 - \alpha) \theta y_{10} + (\alpha - 1) y_8}{2 \gamma_1} (\bar{M} - \bar{M}^*), \]

(A.7)

\[ \ddot{y}^* = \frac{1 + \Re}{(1 + \bar{r})} \bar{M}^w - \frac{\varepsilon y_5 (2 - \alpha) \theta y_{10} + (\alpha - 1) y_8}{2 \gamma_1} (\bar{M} - \bar{M}^*), \]

(A.7*)

\[ \ddot{y} = - \ddot{y}^* = \frac{\varepsilon (1 - \alpha + \alpha \theta) y_5 \gamma_6 \gamma_{12}}{4 \gamma_8 \gamma_{11} \gamma_{13}}, \]

(A.8)

\[ \dot{C} = \frac{1 + \Re}{(1 + \bar{r})} \bar{M}^w + \frac{\varepsilon y_9}{2 \gamma_{11}} (\bar{M} - \bar{M}^*), \]

(A.9)

\[ \dot{C}^* = \frac{1 + \Re}{(1 + \bar{r})} \bar{M}^w - \frac{\varepsilon y_9}{2 \gamma_{11}} (\bar{M} - \bar{M}^*), \]

(A.9*)

\[ \ddot{C} = - \ddot{C}^* = \frac{(1 + \alpha \theta) \varepsilon y_5 \gamma_6 \gamma_{12}}{4 \gamma_8 \gamma_{11} \gamma_{13}}, \]

(A.10)

\[ \ddot{r} = \frac{1 + \Re}{\bar{r} \delta (1 + \bar{r})} \bar{M}^w - (\alpha - 1) \varepsilon y_9 \left( \frac{2 (2 - \alpha) y_{10} \gamma_{13} + \alpha y_{12} \gamma_{13}}{4 (2 - \alpha) \bar{r} \delta y_6 \gamma_{11} \gamma_{13}} \right) (\bar{M} - \bar{M}^*), \]

(A.11)

\[ \ddot{r}^* = \frac{1 + \Re}{\bar{r} \delta (1 + \bar{r})} \bar{M}^w + (\alpha - 1) \varepsilon y_9 \left( \frac{2 (2 - \alpha) y_{10} \gamma_{13} + \alpha y_{12} \gamma_{13}}{4 (2 - \alpha) \bar{r} \delta y_6 \gamma_{11} \gamma_{13}} \right) (\bar{M} - \bar{M}^*). \]

(A.11*)
Nomenclature

\[ \alpha \in (0,2) \]

\[
\begin{align*}
\gamma_0 & = (2-\alpha)(\alpha\theta + 1) + \alpha(\alpha - 1) \\
\gamma_1 & = 1 - \alpha + \varepsilon + \bar{\varepsilon}(2-\alpha) \\
\gamma_2 & = (\varepsilon - 1)(\alpha - 1) \\
\gamma_3 & = 2(1 + \theta - \alpha) \\
\gamma_4 & = (1-\alpha)(2\theta(2-\alpha) - \gamma_5) \\
\gamma_5 & = (2-\alpha)r(\theta + 1) \\
\gamma_6 & = (\alpha - 1)\gamma_5 + (\alpha\theta - 1)\gamma_5 \\
\gamma_7 & = \gamma_5\gamma_1 + \alpha(1-\alpha)\gamma_2 \\
\gamma_8 & = \gamma_6\gamma_4 + \alpha(1-\alpha)\gamma_4 \\
\gamma_9 & = \gamma_6\gamma_4(1 + \bar{r}\varepsilon) \\
\gamma_{10} & = \gamma_6(\gamma_3 + \gamma_4) - \alpha\gamma_4 \\
\gamma_{11} & = \gamma_7\gamma_{10} + \gamma_6 + \alpha\gamma_2\gamma_8 \\
\gamma_{12} & = \gamma_2(\alpha\theta - 1) - \gamma_8 \\
\gamma_{13} & = \theta(1 + \alpha\theta) + (1-\alpha) \\
\end{align*}
\]

\[ \alpha = 1 \]

\[
\begin{align*}
\gamma_0 & = \theta + 1 \\
\gamma_1 & = \varepsilon(1 + \bar{\varepsilon}) \\
\gamma_2 & = 0 \\
\gamma_3 & = 2\theta \\
\gamma_4 & = 0 \\
\gamma_5 & = \bar{r}(\theta + 1) \\
\gamma_6 & = \bar{r}(\theta^2 - 1) \\
\gamma_7 & = \varepsilon(1 + \bar{\varepsilon})(\theta + 1) \\
\gamma_8 & = \bar{r}(\theta + 1)(\theta^2 - 1) \\
\gamma_9 & = \bar{r}(1 + \bar{\varepsilon})\theta + 1)^2(\theta^2 - 1) \\
\gamma_{10} & = (\theta + 1)(\bar{r}(\theta + 1) + 2\theta) \\
\gamma_{11} & = (1 + \bar{r}\varepsilon)(\theta + 1)^2[\varepsilon(\bar{r}(\theta + 1) + 2\theta) + \bar{r}(\theta^2 - 1)] \\
\gamma_{12} & = 2\theta(\theta^2 - 1) \\
\gamma_{13} & = \theta(\theta + 1) \\
\end{align*}
\]

References


